Brain Rhythms and Mathematics

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Oscillations in the human brain

In an EEG, voltages are recorded on a person's scalp.

One gets traces such as this one:

(This is one of the earliest EEG recordings ever made, by Hans Berger in the 1920s.)

EEGs oscillate. Since the 1920s, people have wondered exactly what oscillates in our heads, and why, and what that tells us about how our brains work.

Oscillations have *something* to do with brain function (and also with brain disease)

- When you pay **attention**, you have strong 40 Hz oscillations in certain parts of your brain.
- As you plan a movement, you have strong 20 Hz oscillations in a certain part of your brain; they decrease in amplitude just before movement onset.
- Quiet wakefulness goes together with 10 Hz oscillations.
- **Buddhist meditation** can induce pronounced 40 Hz oscillations in the brain! (Reported in the *Proceedings of the National Academy of Sciences*, 2004.)
- Patients suffering from **Parkinson's disease** have abnormally strong 20 Hz oscillations.
- Patients suffering from **schizophrenia** have abnormally weak 40 Hz oscillations in the auditory cortex.
- **Epileptic seizures** are often preceded by abnormal very high frequency oscillations.
- ...

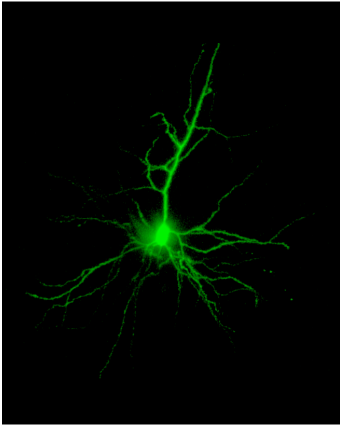
What is a brain?

A human brain consists about a hundred billion nerve cells ("neurons"), together with a probably comparable number of support cells ("glial cells").

To first approximation, one can assume that the neurons do all the information processing, thinking, feeling, etc.

A brief look at individual neurons in the brain

Each of the 100,000,000,000 neurons is, by itself, an object of spectacular complexity and subtlety.



- Inside a neuron, there is a voltage, V, that is typically negative in comparison with the water surrounding the cell. Most of the time, $V \approx -70$ mV or so.
- Neurons can transiently open up channels in their cell membranes to let ions (charged particles) pass through. This can transiently alter V.
- Sometimes V shoots up (to values as high as $+20$ mV or so), then plummets back down (to the usual -70 mV or so), all in the span of 1 or 2 milliseconds. We say then that the neuron has "fired". (Alan Hodgkin and Andrew Huxley got a Nobel Prize in 1963 for a series of four papers, published in 1952, uncovering the biophysical mechanisms causing firing.)

Neuronal communication

- Two kinds of neuronal communication: chemical and electrical. We will focus on chemical communication here.
- When a neuron fires, it releases a neurotransmitter that attaches to neighboring neurons, causing ion channels in their cell membranes to open up.
- About 80% of neurons in the brain are "excitatory" — neurotransmitters that they release tend to *raise* the voltages in neighboring neurons, promoting firing.
- The remaining 20% of neurons are "inhibitory" — the neurotransmitters that they release tend to *lower* voltages in neighboring neurons, counteracting firing.

What oscillates in the brain?

Sometimes massive numbers of neurons in the brain transiently synchronize. For a brief period, they fire rhythmically and in synchrony.

When there are enough of them, the voltage fluctuations are so large that they can be measured on the scalp.

So the oscillations in the EEG reflect rhythmic, synchronized firing of huge, transient coalitions of neurons.

What causes neurons to fire in rhythmic synchrony?

In the 1940's, Norbert Wiener, one of the greatest mathematicians of the 20th century, recognized that this question is fundamentally mathematical in nature.

Norbert Wiener was an alumnus of Tufts College. He is probably the most distinguished intellectual among our alumni. He graduated in 1909, at age 14, and spent most of his adult life as a Professor of Mathematics at MIT.

Norbert Wiener (right) and William Ransom (left)

(William Ransom was a Professor of Mathematics at Tufts.)

From the brain to Calculus

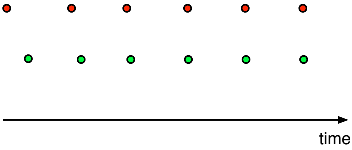
Mathematicians like to simplify, abstracting from complications to a point where clear understanding becomes possible, while capturing essential aspects of reality.

This is a lot like abstract painting!

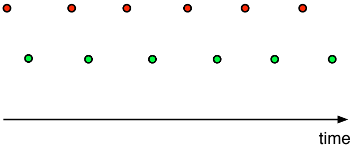
- Don't consider 100,000,000,000 neurons right away... Start with just 2 neurons, call them A and B.
- Assume that A and B, if they don't get input, want to fire periodically (just as the heart beats periodically), with period $= 1$ time unit. In fact, many neurons in the brain fire periodically in the absence of input from other neurons.
- Assume that the interaction between A and B is excitatory: A's firing will cause B to fire earlier, and vice versa.

What exactly does synchronization of two neurons mean?

synchronization:



anti-synchronization:



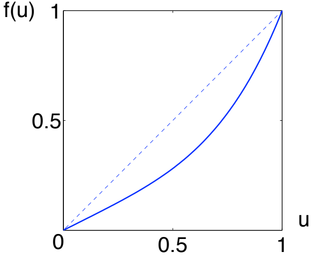
red dots: neuron A fires
green dots: neuron B fires

Mathematical notation

- Suppose neuron B would, in the absence of input from the other neuron, fire u time units from now ($0 \leq u \leq 1$).
- Suppose neuron A fires now. This accelerates the next firing of neuron B. It will now happen $f(u)$ time units from now, where

$$0 \leq f(u) \leq u.$$

  We call f the "interaction function". It might look like this:



- B acts on A the same way: We could reverse the letters "A" and "B" above.

The strobe light idea

Charles S. Peskin, 1975

The key idea that turns our synchronization problem into a Calculus problem:

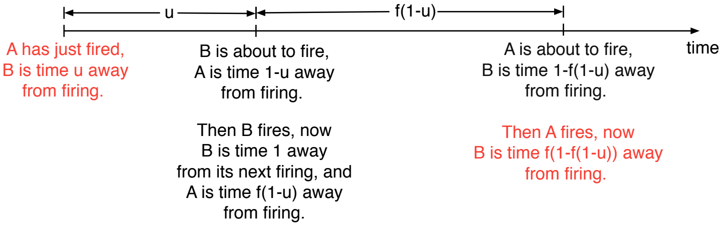
> Focus only on the times when A has just fired.
> At those times, ask: How far is B from firing?

Don't watch the two neurons all the time. Just turn on a "strobe light" at certain times to check what is happening.

From the first firing of A to the second

If A has just fired, and B is time u away from firing, then just after the next firing of A, B will be time $g(u)$ away from firing.

$$g(u) = f(1 - f(1 - u))$$



Iteration

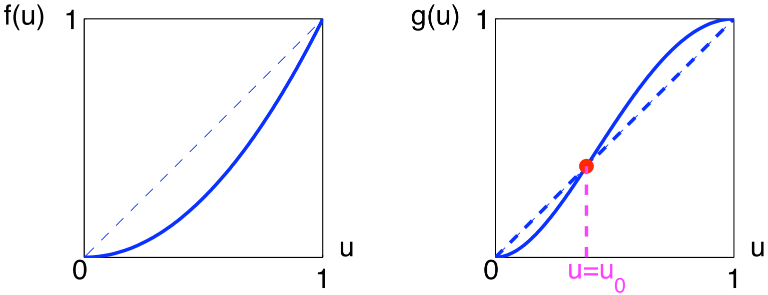
If we watch how far B is from firing every time A has just fired, we see this sequence:

$$u, \quad g(u), \quad g(g(u)), \quad g(g(g(u))), \quad g(g(g(g(u)))), \quad etc.$$

A and B synchronize if this sequence converges

- to 0 (when A fires, B is about to fire), or
- to 1 (when A fires, B has just fired).

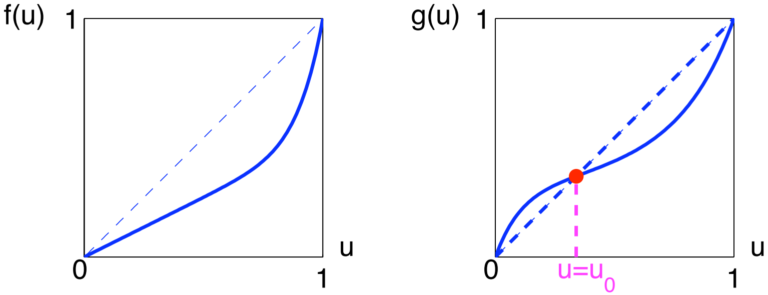
The simplest example: $f(u) = u^2$



If $u < u_0$: $g(u)$ is smaller than u, $g(g(u))$ is even smaller, etc. ... $\to 0$, so synchrony.

If $u > u_0$: $g(u)$ is larger than u, $g(g(u))$ is even larger, etc. ... $\to 1$, so synchrony.

A second example: $f(u) = (u + u^{10})/2$



If $u < u_0$: $g(u)$ is larger than u, $g(g(u))$ is even larger, etc. ... $\to u_0$, so no synchrony.

If $u > u_0$: $g(u)$ is smaller than u, $g(g(u))$ is even smaller, etc. ... $\to u_0$, so again no synchrony.

What we learn from these two examples

Fairly subtle changes in f can make the difference between synchrony and no synchrony!

Put differently: We can't understand synchronization without understanding the biological details.

More to think about

1. Exactly which interaction functions f yield synchrony?
   [I don't fully know.]

2. What if A and B don't have exactly identical properties?

3. What if signals take some time to travel between neurons?
   [A very partial result: For $f(u) = u^2$, short delays prevent synchronization, long delays don't.]

4. What if A *inhibits* B, and vice versa?

5. What happens with more than 2 neurons?

From Calculus back to the brain

- Rhythmic synchrony can *emerge* in the absence of external rhythmic input.
  Norbert Wiener, Tufts' famous alumnus, gave the first mathematical explanation of this point.
- Oscillations in the EEG may arise from synchronization mechanisms similar to the one we have demonstrated, of course operating on a massive scale, with millions of neurons synchronizing.
- Subtle details of the interaction law between two neurons matter for synchrony.
  The plots of $f(u) = u^2$ and $f(u) = (u + u^{10})/2$ don't look all that different, yet one yields synchrony, the other does not.
- The brain can modulate the details of neuronal interaction, for instance using chemicals called neuromodulators.
  The brain might therefore be able to toggle between synchrony and asynchrony using neuromodulators.

Recommended reading (easy and fun!)

Steven Strogatz, *Sync: The Emerging Science of Spontaneous Order*, Hyperion Books, 2003

Advertisement

I do research on problems of this sort. I frequently teach two closely related undergraduate classes:

- Mathematical Neuroscience
- Nonlinear Dynamics and Chaos

The prerequisite for both is Calculus.

Undergraduate students also participate in my research.

Thanks for listening!