Is ranked choice voting a good idea?

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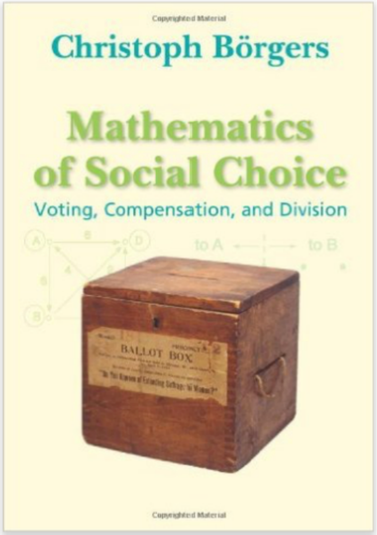
Applications of Mathematics in Political Science studied in the Tufts Mathematics Department

Traditionally, mathematics plays a much smaller role in Political Science than in (for instance) Physics.

Our department is a bit unusual:

- Professor Bruce Boghosian studies the origins of wealth inequality using mathematical methods originally developed for studying gas dynamics.
- Professor Moon Duchin studies gerrymandering of electoral districts using geometry and statistics.
- I have had an interest in the fairness of election methods.

Shameless commerce



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I. What's wrong with the normal way of voting?

Answer: There is no "normal" way of voting.

You have arrived on the Tufts campus, and joined a student club that has 18 members.

The club wants to elect a President. There are four candidates: A, B, C, and D.

All 18 club members vote, and here is the outcome:

A: 6 votes
B: 5 votes
C: 4 votes
D: 3 votes

President A has been elected.

This is how we often vote in the United States.
It's called plurality voting.

A: 6 votes
B: 5 votes
C: 4 votes
D: 3 votes

However, B is unhappy. She argues that most C- and D-voters would have preferred her to A. The club decides to conduct a runoff election between A and B. The outcome:

A: 6 votes
B: 12 votes

So President B has been elected.

This is how the French elect their President.
It's called runoff voting.

A: 6 votes
B: 5 votes
C: 4 votes
D: 3 votes

A: 6 votes
B: 12 votes

But now C objects: Why not have a three-way runoff, C says?

Okay, the club has a three-way runoff involving A, B, and C:

A: 6 votes
B: 5 votes
C: 7 votes

After this, the matter is decided by a runoff between A and C:

A: 6 votes
C: 12 votes

Confusingly, now C seems to be the rightful winner.

This is how the state of Maine now conducts most elections.
I call it the elimination method.

There was a ballot initiative in Massachusetts last November to introduce the elimination method ("ranked choice voting").

The initiative failed. However, in Maine and in New York City, the elimination method has recently been introduced (for most elections).

How could there have been such disparate outcomes with different ways of conducting the vote? To understand this, let's ask all club members to *rank* the candidates. The outcome looks like this:

6	5	4	3
A	B	C	D
B	C	B	C
C	A	A	B
D	D	D	A

Can you see that in fact, plurality voting, runoff, and instant runoff result in three different winners here?

6	5	4	3
A	B	C	D
B	C	B	C
C	A	A	B
D	D	D	A

Jean-Charles de Borda, a French mathematician in the 1700s, had his own idea how to determine the winner: Give each candidate 4 points for a 1st place ranking, 3 points for a 2nd place ranking, 2 points for a 3rd place ranking, 1 point for a 4th place ranking.

A: 45 points
B: 56 points
C: 52 points
D: 27 points

So, Borda would have said, B should win.

This is how the MLB Most Valuable Player is elected.
It is called Borda count.

6	5	4	3
A	B	C	D
B	C	B	C
C	A	A	B
D	D	D	A

Another French nobleman and mathematician of the 1700s, Nicolas de Condorcet, had another argument. He said:

> *In a two-person runoff election, B would win no matter whom B is running against. Therefore B should be elected.*

We call B the *Condorcet candidate.*

This method is not in use in political elections, but it is used by countless organizations around the world to conduct their elections. It is called Condorcet-fair voting.

There may not be a Condorcet candidate

3	4	2
A	B	C
B	C	A
C	A	B

A beats B, 5:4
B beats C, 7:2
C beats A, 6:3

This is called Condorcet's paradox. Perfectly rational individuals can create irrational societal preferences.

Condorcet-fair voting is a framework, not a method. There are many Condorcet-fair voting methods. They differ by the tie breaker they use when there is no Condorcet candidate.

6	5	4	3
A	B	C	D
B	C	B	C
C	A	A	B
D	D	D	A

Plurality voting: A wins (United States, mostly)
Runoff voting: B wins (France, Russia)
Elimination voting: C wins (State of Maine, City of New York))
Borda count: B wins (MLB)
Condorcet-fair voting: B wins (IEEE = Institute of Electrial and Electronics Engineers)

Which of these methods seems best to you?

II. How can we think about which method is best?

Formulate fairness criteria, then check which methods satisfy them.

If more than half of all voters place candidate X first, then candidate X should win.

This is called the majority criterion.

Do you agree that this criterion is important?

If a candidate X would win every two-way runoff election, no matter whom they'd run against, then X should win.

In other words, a Condorcet candidate, if there is one, should win. This of course is called the Condorcet criterion.

Does this seem like an important criterion to you?

If a candidate X would lose every two-way runoff election, no matter whom they'd run against, then X should not win.

This is called the Condorcet loser criterion.

This seems pretty obviously desirable, doesn't it?

It seems clear that a method that violates this criterion shouldn't even be considered!

Let me quickly point out that plurality voting violates it:

10	9	8
A	B	C
C	C	B
B	A	A

Here A is both the plurality winner, and a Condorcet loser (would lose every one-on-one race by a landslide).

If a few voters change their mind and move X up in their rankings, this should never turn X from a winner into a loser.

This is called the monotonicity criterion.

Do you find this important?

Many seemingly reasonable methods violate it, for instance the elimination method.

8	4	12	7	5
A	A	B	C	C
B	C	A	B	A
C	B	C	A	B

Let's look at the records of candidates A and B here:

A: 12 first places, 17 second places, 7 third places
B: 12 first places, 15 second places, 9 third places

We say that the record of A is "objectively better" than that of B.

> If X has an "objectively better" record than Y, then it should be impossible that X loses but Y wins.

I'll call this the comparison criterion.

It would seem a bit unfair if that weren't true, wouldn't it?

25 theorems in one table:

	maj.	Cond.	Cond. loser	monoton.	comp.
plurality	✓	X	X	✓	✓
runoff	✓	X	✓	X	X
elimination	✓	X	✓	X	X
Borda count	X	X	✓	✓	✓
Condorcet-fair	✓	✓	✓ *	✓ *	X

* depends on the tie breaker used when there is no Condorcet candidate

To give you a sense for the math involved here, I'll prove a couple of the 25 theorems to you.

Theorem 18: Borda count satisfies the Condorcet loser criterion.

That is, a Condorcet loser (somebody who'd lose any two-way race) cannot win by Borda count.

	maj.	Cond.	Cond. loser	monoton.	comp.
plurality	✓	X	X	✓	✓
runoff	✓	X	✓	X	X
elimination	✓	X	✓	X	X
Borda count	X	X	✓	✓	✓
Condorcet-fair	✓	✓	✓ *	✓ *	X

I'll give you the proof for 18 voters and 4 candidates. (It works the same with general numbers.)

Each voter gives out $4 + 3 + 2 + 1 = 10$ Borda points, so the total number of Borda points is 180.

The average number of Borda points per candidate is $180/4 = 45$.

If you were placed first by all voters, you'd get $18 \times 4 = 72$ Borda points.

Each time a voter places another candidate above you, you lose one of those 72 Borda points. If you are a Condorcet loser, then each of your 3 rivals gets placed above you by at least 10 voters. Therefore a Condorcet loser can at most get $72 - 10 \times 3 = 42$ Borda points — less than average and therefore not enough to win by Borda count.

QED

Theorem 25: There is no Condorcet-fair method that satisfies the comparison criterion.

	maj.	Cond.	Cond. loser	monoton.	comp.
plurality	✓	X	X	✓	✓
runoff	✓	X	✓	X	X
elimination	✓	X	✓	X	X
Borda count	X	X	✓	✓	✓
Condorcet-fair	✓	✓	✓ *	✓ *	X

To prove this, it's enough to give a single example in which a Condorcet candidate has an objectively worse record than a competing candidate.

8	4	12	7	5
A	A	B	C	C
B	C	A	B	A
C	B	C	A	B

Here B is the Condorcet candidate. But objectively A has the better record:

A: 12 first places, 17 second places, 7 third places
B: 12 first places, 15 second places, 9 third places

QED

In summary, all voting methods have flaws.

	maj.	Cond.	Cond. loser	monoton.	comp.
plurality	✓	X	X	✓	✓
runoff	✓	X	✓	X	X
elimination	✓	X	✓	X	X
Borda count	X	X	✓	✓	✓
Condorcet-fair	✓	✓	✓ *	✓ *	X

Elimination (the method that people call "ranked choice voting") seems pretty bad when you look at this table.

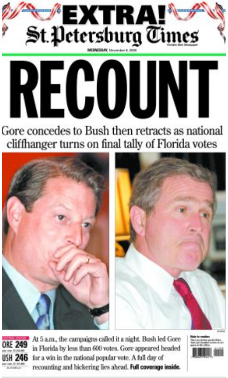
Condorcet-fair methods seem better, but they are never perfect, and cannot be by "Theorem 25".

III. In real life, does it matter?

George W. Bush would not have been President if Florida had used either the elimination method, or a Condorcet-fair voting method.

2020 Presidential election in Florida:

- George W. Bush (Republican): 48.847%
- Al Gore (Democrat): 48.838%
- Ralph Nader (Green): 1.635%



EXTRA!

St. Petersburg Times

RECOUNT

Gore concedes to Bush then retracts as national cliffhanger turns on final tally of Florida votes

ORE 249

USH 246

At 5 a.m., the campaigns called it a night. Bush led Gore in Florida by less than 600 votes. Gore appeared headed for a win in the national popular vote. A full day of recounting and bickering lies ahead. **Full coverage inside.**

The election turned on Florida. Bush won Florida with a margin of 537 votes, according to the official final tally.

The Progressive candidate would not have won the 2009 mayoral election in Burlington, VT if the method used (the elimination method) had been Condorcet-fair.

The three main candidates were Bob Kiss (Progressive), Kurt Wright (Republican), Andy Montroll (Democrat).

The preferences were approximately like this:

1560	994	2327	655	1140	2158
M	M	K	K	W	W
K	W	M	W	K	M
W	K	W	M	M	K

Montroll was the Condorcet candidate, but Kiss won.
Most of Wright's supporters preferred Montroll to Kiss, but managed to get their least favorite candidate elected by voting honestly.

Donald Trump would probably not have been elected in 2016 if the Republicans had used a Condorcet-fair method in their primaries.

This is less certain than the previous two examples, but it seems likely that Donald Trump was strongly liked by a good number of primary voters, and, initially at least, strongly disliked by an even greater number. He was *polarizing.*

Polarizing candidates tend to win under plurality voting (and can win even under elimination voting, as the Burlington example shows), but not under Condorcet-fair voting.

The New York Times

SundayReview | OPINION

How Majority Rule Might Have Stopped Donald Trump

By ERIC MASKIN and AMARTYA SEN APRIL 28, 2016

Summary

1. many different ways of electing a single winner
2. many fairness criteria
3. provably, no method can satisfy them all

What to do?

- decide that some fairness criteria aren't so important to us
- try to construct methods that make violations of important fairness criteria very unlikely, even though not impossible

I am working on the second of these points with an undergraduate research student.

Thank you!

I would be happy to hear from you: cborgers@tufts.edu and even happier to see you in the fall.